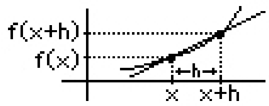
2a (11-97)

Calcu-List



$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Vertical Motion in Feet and Seconds:

$a(t) = v'(t) = h''(t)$

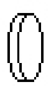
$h(t) = -16t^2 + V_0 t + H_0$

$v(t) = -32t + V_0$

$a(t) = -32$

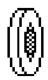
Volumes of Rotation

Discs



$\pi \int r^2 \, dx$

Washers



$\pi \int R^2 - r^2 \, dx$

Shells

$2\pi \int r\,h \, dx$

The Fundamental Theorem of Calculus!

If $f'(x)$ is continuous from a to b then:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

If $f(x)$ is continuous from a to b then:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} \quad \text{or} \quad \frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$$

Graphing Tips

$\lim_{x\to\pm\infty} f(x) = c$ → Horizontal Asymptote at $y = c$

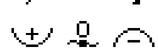
$\lim_{x\to\pm\infty} f(x) = cx$ → Slant Asymptote with slope c

f(undefined value) $= \frac{c}{0}$ → Vertical Asymptote

f(undefined value) $= \frac{0}{0}$ → Hole in the graph

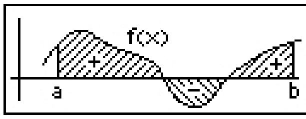
$y' =$ slope →

$y'' =$ concavity →



$y' = 0$ or $\varnothing$ → Indicates possible Max or Min

$y'' = 0$ or $\varnothing$ → Indicates possible Inflection Point



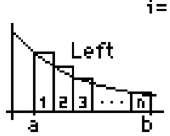
Net Area $= \int_a^b f(x)\,dx$

Trapezoidal Rule (n is the number of trapezoids)

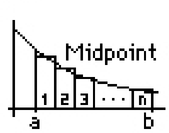
$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + f(x_n)\right)$$

Approximate Area Using Rectangles of Equal Width

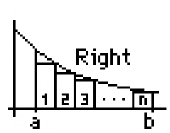
$$\text{Area} = \sum_{i=1}^{n} f(c_i)\cdot\Delta x \qquad \Delta x = \frac{b-a}{n}$$



$c_i = a + (i-1)\cdot\Delta x$



$c_i = a + (i - \frac{1}{2})\cdot\Delta x$



$c_i = a + i\cdot\Delta x$

Derivatives

$\frac{d}{dx}(ax^n) = nax^{n-1}$

$\frac{d}{dx}(uv) = u'v + uv'$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\text{arccot}\, x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\text{arcsec}\, x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\text{arccsc}\, x) = \frac{-1}{|x|\sqrt{x^2-1}}$

Integrals

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

$\int e^x \, dx = e^x + c$

$\int \frac{1}{x} \, dx = \ln|x| + c$

$\int \sin x \, dx = -\cos x + c$

$\int \cos x \, dx = \sin x + c$

$\int \tan x \, dx = -\ln|\cos x| + c$

$\int \cot x \, dx = \ln|\sin x| + c$

$\int \sec x \, dx = -\ln|\sec x - \tan x| + c$

$\int \csc x \, dx = \ln|\csc x - \cot x| + c$

$\int \sec^2 x \, dx = \tan x + c$

$\int \csc^2 x \, dx = -\cot x + c$

$\int \sec x \tan x \, dx = \sec x + c$

$\int \csc x \cot x \, dx = -\csc x + c$

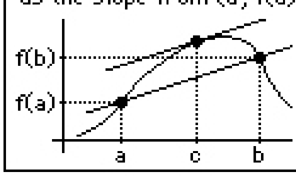
$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + c$

$\int \frac{1}{1+x^2} \, dx = \arctan x + c$

$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \text{arcsec}|x| + c$

The Mean Value Theorem

If $f(x)$ is continuous and differentiable from a to b, then there is a x-value c such that the slope at c is the same as the slope from $(a, f(a))$ to $(b, f(b))$.



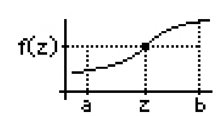
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Average Value

From a to b on a continuous $f(x)$ there is a z such that:

- At z, $f(x)$ takes on the average value.
- $f(z)$ is the average value.

$$f(z) = \frac{\int_a^b f(x)\,dx}{b-a}$$



Separable Differential Equations -- Exponential Growth

When y is directly proportional to the rate at which y changes: $\Rightarrow \frac{dy}{dt} = ry$

$$\Rightarrow \frac{1}{y}dy = r\,dt \Rightarrow \int \frac{1}{y}dy = \int r\,dt \Rightarrow \ln y = rt + c$$

$$\Rightarrow e^{\ln y} = e^{rt+c} \Rightarrow y = e^{rt}\cdot e^{c} \Rightarrow \boxed{y = pe^{rt}}$$